Conditional Logic Explanation

$$\boldsymbol{p} \rightarrow \boldsymbol{q} : \textit{The Conditional Connective}$$
$$\boldsymbol{q} \rightarrow \boldsymbol{p} : \textit{The Converse Conditional Connective}$$
$$\boldsymbol{p} \leftrightarrow \boldsymbol{q} : \textit{The Biconditional Connective}$$

This Supplemental Information will help you to understand why the differences between a conditional proposition, converse of the conditional proposition, and the biconditional proposition.

First, we show a little vocabulary to ensure we are speaking the same language

Vocabulary (Michaels, Rosen, Gross, Grossman, & Shier, 1999):

1. In English, the conditional connective $p \rightarrow q$ represents the following:
   - *if p then q*
   - *q if p*
   - *p only if q*
   - *p implies q*
   - *q follows from p*
   - *q whenever p*
   - *p is a sufficient condition for q*
   - *q is a necessary condition for p.*

2. In English, the biconditional connective $p \leftrightarrow q$ represents the following:
   - p if and only if q (often written p iff q)
   - p and q imply each other
   - p is a necessary and sufficient condition for q
   - p and q are equivalent.

3. Computer programming uses the following notation for logical operators:
   - *p* AND *q* for $p \wedge q$
   - *p* OR *q* for $p \vee q$
   - NOT *p* for $\neg p$
   - *p* XOR *q* for $p \oplus q$
   - *p* NOR *q* for $p \downarrow q$
   - *p* NAND *q* for $p \mid q$.

4. To solve the issue, what is a converse of a conditional?

The converse of the conditional $p \to q$ is the proposition $q \to p$.

What we must know about conditionals in propositional logic:

Conditionals are compound sentences, sentences that contain another sentence as one of its parts. One sentence acts as the antecedent, which states the condition, while the other sentence acts is the consequent, which depends on the condition. Sometimes even the antecedent and the consequents themselves can be compound sentences.

In this course, we break down the language into Standard English, and base the validity, true or false, on the argument form. This allows us concentrate on the logic alone. The truth then depends on whether the antecedent stands in the appropriate relationship to the situation described in the consequent. When we convert the language symbolically, we focus only on the truth-functional relationship of the conditional. (Salmon, 2007).

In summary, for the purpose of Sentential Logic, the truth or falsity of a conditional sentence depends on the truth of falsity of its parts (Salmon, 2007).

The Conditional

The Problem (Discrete Mathematics Seventh Edition, 2009, p. 25):

Step 1a: Write the conditional proposition:

If Jerry receives a scholarship, then he will go to college.

Step 2a: Show symbolically the conditional problem:

Let

$p = \ Jerry\ recieves\ a\ scholarship$

$q = \ Jerry\ goes\ to\ college.$

Written symbolically we have: $p \to q$

Step 3a: Build a Truth Table:

	p	q	p → q
Case 1	T	T	T = $If\ Jerry\ recieves\ a\ scholarship, then\ Jerry\ goes\ to\ college.$
Case 2	T	F	F = $If\ Jerry\ recieves\ a\ scholarship, then\ Jerry\ does\ not\ go\ to\ college.$
Case 3	F	T	T = $If\ Jerry\ does\ not\ recieve\ a\ scholarship, then\ Jerry\ goes\ to\ college.$
Case 4	F	F	T = $If\ Jerry\ does\ not\ recieve\ a\ scholarship, then\ Jerry\ does\ not\ go\ to\ college.$

Case 1: The conditional sentence is true. The antecedent (*p*) stands in the appropriate relationship with the consequent (*q*).

Case 2: This conditional sentence is false. The consequent (q) is not supporting the antecedent. We create an invalid conditional argument.

Case 3: The conditional sentence is true. The consequent (q) can still support a false antecedent (*p*). As Johnsonbaugh, the course textbook author explains on page 25, Jerry could win the lottery, and still go to college without a scholarship. The consequent does not need a true antecedent to make a true argument.

Case 4 The conditional sentence is true. The consequent still supports the antecedent even though it is false. Therefore the argument is true.

Therefore: The only way in which a conditional sentence is not true is for it to have a true antecedent (*p*) and a false consequent (*q*).

The Converse Conditional

Step 1b: Write the converse conditional proposition:

If Jerry goes to college, then Jerry receives a scholarship.

Step 2b: Show symbolically the converse conditional problem:

Let

$$p = \ Jerry\ recieves\ a\ scholarship$$
$$q = \ Jerry\ goes\ to\ college.$$

Written symbolically we have: $q \rightarrow p$

Step 3b: Build a Truth Table:

	p	q	q → p
Case 1	T	T	T = $If\ Jerry\ goes\ to\ college, then\ Jerry\ recieves\ a\ scholarship.$
Case 2	T	F	F = $If\ Jerry\ goes\ to\ college, then\ Jerry\ does\ not\ recieve\ a\ scholarship.$
Case 3	F	T	F = $If\ Jerry\ does\ not\ go\ to\ college, then\ Jerry\ recieves\ a\ scholarship.$
Case 4	F	F	T = $If\ Jerry\ does\ not\ go\ to\ college, then\ Jerry\ does\ not\ recieve\ a\ scholarship.$

Case 1: The conditional sentence is true. The antecedent (*p*) stands in the appropriate relationship with the consequent (*q*).
Case 2: This conditional sentence is false. The consequent (q) is not supporting the antecedent. We create an invalid conditional argument.
Case 3: This conditional sentence is false. The consequent (q) is not supporting the antecedent. We create an invalid conditional argument.
Case 4 The conditional sentence is true. The antecedent (*p*) stands in the appropriate relationship with the consequent (*q*).
Therefore: The only way in which a converse conditional sentence is true is for it to have the same truth values for the antecedent (*p*) and the consequent (*q*).

The Biconditional

Now that we understand how a conditional works, we look at the biconditional (↔).

We noticed when working with conditionals that the consequent (*q*) must support the antecedent (*a*). A biconditional requires that the antecedent (p) support the consequent (q) AND the consequent (q) supports the antecedent (a). This is sometimes called a material biconditional, and can be symbolically represented as:

$$(\boldsymbol{p} \rightarrow \boldsymbol{q}) \wedge (\boldsymbol{q} \rightarrow \boldsymbol{p}) or\ more\ commonly\ \boldsymbol{p} \leftrightarrow \boldsymbol{q}$$

To illustrate the difference of condtional vs. biconditional, we re-examine the example:

If and only if Jerry receives a scholarship, Jerry goes to college.

Step 1c: Write the biconditional proposition:

If and only if Jerry receives a scholarship, Jerry goes to college.

Step 2c: Show symbolically the converse conditional problem:

Let

$p = \ Jerry\ recieves\ a\ scholarship$

$q = \ Jerry\ goes\ to\ college.$

Written symbolically we have: $p \leftrightarrow q$

Step 3c: Build a Truth Table: *(Iff = If and only if)*

p	**q**	**p ↔ q**
T	**T**	**T =** $Iff\ Jerry\ receives\ a\ scholarship, Jerry\ goes\ to\ college.$
T	**F**	**F =** $Iff\ Jerry\ receives\ a\ scholarship, Jerry\ does\ not\ go\ to\ college$
F	**T**	**F =** $Iff\ Jerry\ does\ not\ receive\ a\ scholarship, Jerry\ goes\ to\ college$
F	**F**	**T =** $Iff\ Jerry\ does\ not\ receive\ a\ scholarship, Jerry\ does\ not\ go\ to\ college$

We know as an "If and only if" statement both must be true. If Jerry goes to college, Jerry received a scholarship. If Jerry receives a scholarship, Jerry goes to college. The interdependence is on both the antecedent (*p*) and the consequent (*q*).

To conclude this supplemental study we offer the following quoted text from the course room textbook (Discrete Mathematics Seventh Edition, 2009, p. 29):

> "In formal logic, "if" and "if and only if" are quite different. The conditional proposition $p \rightarrow q$ is true except when *p* is true and *q* is false. On the other hand, the biconditional proposition $p \leftrightarrow q$ is true precisely when *p* and *q* are both true and false."

References

Johnsonbaugh, R. (2009). *Discrete Mathematics Seventh Edition.* Upper Saddle River, NewJersey: Prentice Hall.

Michaels, J. G., Rosen, K. H., Gross, J. L., Grossman, J. W., & Shier, D. R. (1999). *Handbook of Discrete and Combinatorial Mathematics.* New York: CRC Press.

Salmon, M. H. (2007). *Introduction to Logic and Critical Thinking.* Belmont: Thomson Wadsworth.