



MAT2051 Discrete Mathematics

Lecture and Tutorial on Logical Operators

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Trouble with Truth Tables

- There are a fair number of rules to remember
- There are two ways to build Truth Tables
 - Manually
 - Using Excel Logical Operators
- There are six basic Truth Tables to remember
 - AND \wedge
 - OR \vee
 - NOT \neg
 - EXCLUSIVE OR \oplus
 - CONDITIONAL (IMPLIES) \rightarrow
 - BICONDITIONAL \leftrightarrow



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MANUAL TRUTH TABLES



How to Begin

Begin your truth tables the same systematic way

Two Propositions

TT, TF, FT, FF

p	q
T	T
T	F
F	T
F	F

Three Propositions

TTT, TTF, TFT, TFF, FTT, FTF, FFT, FFF

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F



AND Truth Table

Truth Table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Key Points

- Just one **TRUE** row!

Both premises must be true, for the conclusion to be true.



OR Truth Table

Truth Table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Key Points

- Three **TRUE** rows!

Both premises must be false, for the conclusion to be false.



NOT Truth Table

Truth Table

p	$\neg p$
T	F
T	F
F	T
F	T

Key Points

- **T** and **F** become opposites



EXCLUSIVE OR Truth Table

Truth Table

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Key Points

- Two **TRUE** rows!

Both true rows are in the middle.



CONDITIONAL (IMPLIES) Truth Table

Truth Table

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Key Points

- Three **TRUE** rows!
- Second row **False**!

When the consequent is **FALSE**, the antecedent must be **FALSE** too for the proposition to be **TRUE**.



BICONDITIONAL Truth Table

Truth Table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Key Points

- Two **TRUE** rows!
- Middle rows **False**!

The consequent and antecedent must both be the **SAME** for the proposition to be **TRUE**.



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LOGICAL OPERATORS IN EXCEL



Using Microsoft Excel

- Excel can perform a lot of mathematical operations and calculations for us.
- Unfortunately, Excel only has three of the six basic logical operators, AND, OR, NOT.
- But we can build the missing three with the basic three.



Three Logical Operators

The three logical operator/functions that allow us construct Truth Tables in Excel:

- =OR(): The **OR** function
- =AND(): The **AND** function
- =NOT(): The **NOT** function

Note:

- Both the OR and AND functions take two parameters TRUE or FALSE
=OR(TRUE,FALSE).
- The NOT function takes only one parameter
=NOT(TRUE).



In building Truth Tables, write out TRUE and FALSE instead of T and F. Here's the basic three in operation:

	A	B	C	D	E	F	G	H	I	J	K	L
16												
17		<u>p</u>	<u>q</u>	<u>OR(p,q)</u>	<u>AND(p,q)</u>	<u>NOT(p)</u>						
18		TRUE	TRUE	TRUE	TRUE	FALSE						
19		TRUE	FALSE	TRUE	FALSE	FALSE						
20		FALSE	TRUE	TRUE	FALSE	TRUE						
21		FALSE	FALSE	FALSE	FALSE	TRUE						
22												
23												
24												
25												
26												
27												
28												
29												
30												

Formulas:

OR(p,q), cell D18: =OR(B18,C18)
...
OR(p,q), cell D21: =OR(B21,C21)

AND(p,q), cell E18: =AND(B18,C18)
...
AND(p,q), cell E21: =AND(B21,C21)

NOT(p), cell F18: =NOT(B18)
...
NOT(p), cell F21: =NOT(B21)

For instance, for the OR operator, all you have to do is type =OR (then point to the first cell, type a comma, then point to the second cell, then type) Enter.

Once you have the first formula typed, you can use the fill handle to copy the formula down the rest of the column. Excel fills in the answer.

The most important logical operator Excel does not provide is the IMPLIES or CONDITIONAL operator, but fortunately, we have a fairly easy logical equivalent:

$$p \rightarrow q \equiv \neg p \vee q$$

In Excel, we write this as =OR(NOT(p),q).

E11		fx		=OR(NOT(B11),C11)					
	A	B	C	D	E	F	G	H	
1									
9	Logical Equivalence of IMPLIES (CONDITIONAL) Operator:								
10		p	q	P-->q	OR(NOT(p),q)				
11		TRUE	TRUE	TRUE	TRUE	Formula: cell E11: =OR(NOT(B11),C11)			
12		TRUE	FALSE	FALSE	FALSE				
13		FALSE	TRUE	TRUE	TRUE				
14		FALSE	FALSE	TRUE	TRUE				
15									



The EXCLUSIVE OR and BICONDITIONAL logical equivalences are not as easy. The EXCLUSIVE OR :

$$p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$$

In Excel, we write this as =OR(AND(p,NOT(q)),AND(NOT(p),q))

E19					
fx =OR(AND(B19,NOT(C19)),AND(NOT(B19),C19))					
	A	B	C	D	E
1					
17	Logical Equivalence of Exclusive OR Operator:				
18		p	q	P XOR q	OR(AND(p,NOT(q)),AND(NOT(p),q))
19		TRUE	TRUE	FALSE	FALSE
20		TRUE	FALSE	TRUE	TRUE
21		FALSE	TRUE	TRUE	TRUE
22		FALSE	FALSE	FALSE	FALSE
23					
24					
25					
26					
27					
28					

Formula:

cell E19:
=OR(AND(B19,NOT(C19)),AND(NOT(B19),C19))



The BICONDITIONAL operator has a similar logical equivalence

$$p \leftrightarrow q \equiv (\neg p \vee q) \wedge (p \vee \neg q)$$

In Excel, we write this as =AND(OR(NOT(p),q),OR(p,NOT(q)))

E33		fx =AND(OR(NOT(B33),C33),OR(B33,NOT(C33)))				
	A	B	C	D	E	F
1						
31	Logical Equivalence of BICONDITIONAL Operator:					
32		p	q	P IFF q	AND(OR(NOT(p),q),OR(p,NOT(q)))	
33		TRUE	TRUE	TRUE	TRUE	
34		TRUE	FALSE	FALSE	FALSE	
35		FALSE	TRUE	FALSE	FALSE	
36		FALSE	FALSE	TRUE	TRUE	
37						
38						
39						
40						
41						
42						

Formula:
cell E33:
=OR(AND(B19,NOT(C19)),AND(NOT(B19),C19))



Now, you not only know how to do all logical operations in Excel, you also know how to do logical equivalences!