**Unit 3**

**Problem 6.3.9**

**Question:** *How many strings can be formed by ordering the letters SCHOOL using  
some or all of the letters?*

What do we know?

* The problem indicates “How many strings” Therefore, we cannot have repetitions, as strings do not allow repetitions. We can denote repetitions in strings such as denoting SCHOOL as SCHO2L since there are two O’s, but we cannot have two strings of SCHOL, SCHOL using the two O’s as they use the same characters and thus are the same string.
* We can order the Letters SCHOOL in any order, so order does not matter, but since a string is a sequence, order is taken into account to define a string as separate from a set. For example, SCHOL is different than CHOLS.
* The word SCHOOL is 6 characters long and the directions indicate using “*some or all of the letters*” Therefore, we can start with strings of length zero through strings of length 6.

We will break this problem down into multiple parts by string lengths. Then we will add the answers for each string length to get our total.

* **Length 6:** *How many strings of length 6 can be formed by ordering the letters*

*SCHOOL using some or all of the letters?*

* **Length 5:** *How many strings of length 5 can be formed by ordering the letters*

*SCHOOL using some or all of the letters?*

* **Length 4:** *How many strings of length 4 can be formed by ordering the letters*

*SCHOOL using some or all of the letters?*

* **Length 3:** *How many strings of length 3 can be formed by ordering the letters*

*SCHOOL using some or all of the letters?*

* **Length 2:** *How many strings of length 2 can be formed by ordering the letters*

*SCHOOL using some or all of the letters?*

* **Length 1:** *How many strings of length 1 can be formed by ordering the letters*

*SCHOOL using some or all of the letters?*

* **Length 0:** *How many strings of length 0 can be formed by ordering the letters*

*SCHOOL using some or all of the letters?*

We will start with strings of length 0 and work up to strings of length 6.

**Length 0:** *How many strings of length 0 can be formed by ordering the letters*

*SCHOOL using some or all of the letters?*

The string with no elements is called the null string and contains  
exactly one element. Therefore, we have 1 string the null set.

**Total Strings of Length 0 = 1**

**Length 1:** *How many strings of length 1 can be formed by ordering the letters  
SCHOOL using some or all of the letters?*

For strings of length 1 we have S, C, H, O, L = 5  
We cannot count the repeated O since it is the same character.

**Total Strings of Length 1 = 5**

**Length 2:** *How many strings of length 2 can be formed by ordering the letters*

*SCHOOL using some or all of the letters?*

Looking at the two OO’s, this string is made up of the same two  
characters and can be denoted as O2. We count it only as one string of length two rather than two strings because the string OO is no different than the string OO.

Removing the two OO’s we are left with SCHOL. Since we can only pick two characters for a string of length two, and we have five characters to chose from, we have 5\*4 = 20. Thus, we have 20 + 1 = 21 strings.

**Total Strings of Length 2 = 21**

**Length 3:** *How many strings of length 3 can be formed by ordering the letters*

*SCHOOL using some or all of the letters?*

Let’s first address the two OO’s… With strings of length three, with  
two O’s together we have four letters that can be used in three places. Before the O’s, after the O’s or between the O’s. For example, SOO, OOS, and OSO. Thus, we have 4\*3 = 12 strings of length 3 using both O’s.

Now left with the characters SCHOL, we have 5 \* 4 \* 3 ways to select strings of length 3. Thus, we have 5\*4\*3 = 60 plus the 12 strings using both O’s is 72 strings of length three.

**Total Strings of Length 3 = 72**

**Length 4:** *How many strings of length 4 can be formed by ordering the letters*

*SCHOOL using some or all of the letters?*

First addressing the two OO’s… Since the O’s make up two characters out of a possible length of 4, we have {(x), (y), (OO)} -three elements – which gives 3! ways to permute three elements and 4\*3 ways to permute {S,C,H,L} within this permutation. (4\*3) \* (3!) = 12 \* 6 =72

Now considering the letters {S, C, H, O, L} we have 5\*4\*3\*2 = 120 that do not use the two OO’s. Summing these values 120 + 72 = 192

**Total Strings of Length 4 = 192**

**Length 5:** *How many strings of length 5 can be formed by ordering the letters*

*SCHOOL using some or all of the letters?*

Addressing the two OO’s … we have to chose three letters out of the four letters {S, C, H, L} to go with the two OO’s to make a string of length 5, thus we have C(4,3). There will be five letters with two repeated OO’s, therefore we will have 5!/2! ways to permute these letters. C(4,3) \* 5!/2! = 4 \* 60 = 240 strings of length 5.

Now considering the letters {S, C, H, O, L} we have 5! = 120 that do not use the two OO’s. Summing these values 240 + 120 = 360

**Total Strings of Length 5 = 360**

**Length 6:** *How many strings of length 6 can be formed by ordering the letters*

*SCHOOL using some or all of the letters?*

Finally, an easy calculation. 6!/2! = 720/2 = 360.

**Total Strings of Length 6 = 360**

**Now we can sum up all of our calculations:**

Total Strings of Length 0 = 1

Total Strings of Length 1 = 5

Total Strings of Length 2 = 21

Total Strings of Length 3 = 72

Total Strings of Length 4 = 192

Total Strings of Length 5 = 360

Total Strings of Length 6 = 360

**Total of all strings = 1 + 5 + 21 + 72 +192 + 360 + 360 = 1011**