A Little resolution for Resolution!

Undoubtedly, resolution is a little difficult to grasp at first. Note resolution is based on the rule of resolution (page 90 top of page).

For simplicity let me declare the following use of logical operators:

! == Not
&& == Logical And
|| == Logical Or
--> == Conditional
<--> == Biconditional

So first we state the **Rule of Resolution**:
If p || q and !p|| r are both true, then q || r is true.

We can verify this drawing the truth table:
First we label our variables p, q, r which means we have 2^3 or 8 rows. Next we isolate steps of the problem so that we can see what the truth table tells us. We label p or q as step 1, not p or r as step 2, if (step1) then (step2) as step 3, and finally q or r as step 4. What we should find is that when steps 1 and 2 are true, then steps 3 and 4 are true as well.

Variables			**Step 1**	**Step 2**	**Step 3**	**Step 4**
p	q	r	p || q	!p || r	(Step1) --> (Step2)	q || r
1	1	1	1	1	1	1
1	1	0	1	0	0	1
1	0	1	1	1	1	1
1	0	0	1	0	0	0
0	1	1	1	1	1	1
0	1	0	1	1	1	1
0	0	1	0	1	1	1
0	0	0	0	1	1	0

Now that we proved the resolution rule in the truth table above, we can apply the Rule of Resolution repeatedly to statement making new statements in search of a conclusion. Sometimes we run into a situation where we do not have a clear path or clause to change out problem into or expressions. In this case we can use DeMorgan's Laws to find a logically equivalent expression we can work with.

DeMorgan's Laws (applicable to resolution):

DeMorgan Law A: !(p || q) is logically equivalent to !p && !q
DeMorgan Law B: !(p && q) is logically equivalent to !p || !q
DeMorgan Law C: p || qr is logically equivalent to (p || q)(p || r)

To construct a proof by resolution, we replace anything that is not a clause, with as many clauses it takes to construct a logically equivalent argument. We then place the hypothesis in the form of the resolution

rule (or statements) until we derive the conclusion. In some instance we may have to combine with proof by contradiction, proving a known false to show it is true.

To illustrate how this works, we look at a similar problem (*problem 5 from the textbook*).

Hypothesis 1 = p --> q
Hypothesis 2 = p || q
Conclusion = therefore q

Since Hypothesis 2 is already an **or** statement, we only need to concentrate on Hypothesis 1. To change Hypothesis 1 we apply the ***Rule of Resolution***. Let us clarify what we mean by apply, as the textbook makes this a little vague. Look at the ***truth table*** that we made when we verified the ***Rule of Resolution***. We notice that Step 2 and Step 3 are logically equivalent. Therefore if !p || r is logically equivalent to Step 3 when we have three variables (p,q,r) we test to see if when we have only two variables (p,q) is p implies q logically equivalent to !p || q. We build a truth table and test it!

Variables		Implies	Or Statement
p	q	p --> q	!p || q
1	1	1	1
1	0	0	0
0	1	1	1
0	0	1	1

We find that they are logically equivalent; their respective truth-values are the same! This is how we apply the Resolution Rule! So we substitute !p || q for p --> q. We now rewrite our argument:

Hypothesis 1 = !p || q
Hypothesis 2 = p || q
Conclusion = therefore q

Cancelling the !p and p we end up with q || q therefore q. We thus derive the conclusion and finish our proof using **Resolution**.

Applying what we learned about this argument, if we were to look at a Biconditional like occurs in 2.3 problem 6, we would have to first change the Biconditional p <--> r into two conditional expressions. Example 1.3.14 and Example 1.3.15 pages 27 and 28 in the textbook plainly illustrate that p <--> r is logically equivalent to p --> r and r --> p. Therefore, 2.3.6 can be rewritten as:

Hypothesis 1 = p --> r
Hypothesis 2 = r --> p
Hypothesis 3 = r
Conclusion = therefore p

Now to solve the problem, just perform the same steps we did in problem 5! Since this is an assignment problem, that is as far as I take it.